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The steady motion of a sphere in a dusty gas

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The paper considers the effect on the steady flow past a sphere of a uniform upstream distribution of dust particles having a small relaxation time. Using a potential solution as an upstream model of the gas flow at large Reynolds numbers R, an equation for the concentration of dust near the sphere is derived and solved numerically. It is shown that in this inviscid model there exists a dust-free layer adjacent to the sphere. A drag force is computed, and it is also shown that particles do not collide with the sphere until the Stokes number σ is greater than $\frac{1}{12}$ if we assume the gas flow unchanged by the presence of dust particles, which is in agreement with previous work of Langmuir & Blodgett (1946). The paper concludes with a discussion of the effect of a viscous boundary layer in which it is suggested that the dust-free layer is preserved when $\sigma R^{\frac{1}{2}} \gg 1$, but is prevented from forming by the viscous boundary layer when $\sigma R^{\frac{1}{2}} \ll 1$.

1. Introduction

Interest in problems of mechanics of systems with more than one phase has developed rapidly in recent years. Situations which occur frequently are concerned with the motion of a liquid or gas which contains a distribution of solid particles. Such situations occur, for example, in the movement of dust laden air, in problems of fluidization, in the use of dust in gas-cooling systems to enhance heat transfer processes, and in the process by which raindrops are formed by the coalescence of small droplets which might be considered as solid particles for the purpose of examining their movement prior to coalescence.

The mathematical description of such diverse systems must of course vary widely. In problems of fluidization, for example, the bulk concentration of particles is large, whereas in problems of dust flow this will be small. This factor may bring considerable simplification to the theory of dusty gas flows, since the effect of one particle on another is not so pronounced, and a good approximation might be expected by assuming the motion of one solid particle not to be influenced by the surrounding particles which will be in general many particle diameters away. The work of this paper is concerned with dusty gas flow under this simplifying assumption.

Much work has already been done on such models of dusty gas flow as, for example, the work of Carrier (1958), Rudinger (1964) and Marble (1962) on shock waves in dusty gases, and the discussion of the Prandtl–Meyer expansion by Marble (1962). The author's interest in this subject was aroused by a paper of Saffman (1962) in which the Orr–Sommerfeld equation for small disturbances in plane parallel flow of a dusty gas was formulated. Using the model described by Saffman, the author considered in more detail the stability of plane Poiseuille flow (Michael 1964).

In this paper the author has studied the steady motion of a sphere in a dusty gas. The dust is represented by a large number density N of small dust particles whose volume concentration is small, but with appreciable mass concentration. It is assumed that the individual particles of dust are so small that a Stokes flow approximation to their motion relative to the gas is appropriate. The equations of motion give rise to two additional independent parameters due to the presence of the dust, which may be taken as the mass concentration of the dust f and a relaxation time τ. The latter parameter is representative of the time scale on which the velocity of the dust adjusts to changes in the neighbouring gas velocity. When $\tau = 0$ this adjustment is instantaneous, and we have a limiting case in which the dust moves with gas at each point. It may be seen that the motion in this case is closely related to the flow of a clean gas. We consider here the flow of a dusty gas for small non-zero values of τ by a perturbation of the solution at $\tau = 0$.

In considering the steady flow past a sphere of radius a, although the relative motion of small dust particles to the gas is taken to be a Stokes flow, i.e. of small Reynolds number based on particle size and relative velocity, the motion of the gas in the large, past the sphere will in many cases of interest occur at large Reynolds numbers based on the length scale a. Here we have chosen to consider this Reynolds number to be large, and as a first step towards the solution the paper considers in detail the perturbation of the unseparated potential flow for a sphere. This has the advantage of mathematical simplicity, and although it is not the precise solution for the irrotational approach to the sphere when separation occurs, it nevertheless has qualitatively the correct form in the neighbourhood of the upstream stagnation point.

The analysis shows that when a non-singular perturbation of a potential flow is assumed the concentration of dust particles becomes logarithmically infinite at the front stagnation point of the sphere. We find also that dust particles cannot reach the sphere except at the front stagnation point, there being a dust streamline emanating from that point which delineates a thin dust-free layer adjacent to the sphere whose thickness is of order σa, where σ is the Stokes number $\tau U/a$, with U the velocity of the sphere.

The paper shows also that a drag force on the sphere can be calculated on the basis of the unseparated potential flow, and we include a short discussion of a related topic which confirms a result previously given by Langmuir & Blodgett (1946), namely that if changes in the gas flow due to the dust particles are neglected dust particles begin to collect on the sphere when $\sigma = \frac{1}{12}$.

The last part of this paper gives a discussion of the modifications to the 'inviscid' solution due to the viscous boundary layer at large Reynolds numbers. It is seen first that the viscosity cuts off the build-up of the concentration of particles near the sphere. We find also the critical parameter in this part of the discussion is $\sigma R^{\frac{1}{2}}$, where R is the Reynolds number Ua/ν, with the gas kinematic viscosity ν. This parameter represents the ratio of the width of the dust-free layer to the width of the viscous boundary layer. When $\sigma R^{\frac{1}{2}} \gg 1$ it is seen that the dust-free layer is not substantially changed, but when $\sigma R^{\frac{1}{2}}$ is not large a consequence of the reduction of the tangential velocity in the boundary layer is that the dust-free layer ceases to exist.

2. Formulation

The equations used to represent the motion of a dusty gas, given by Saffman are the following:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u}.\nabla)\,\mathbf{u}\right) = -\operatorname{grad} p+\mu\nabla^2\mathbf{u}+KN(\mathbf{v}-\mathbf{u}), \tag{1}$$

$$\operatorname{div}\mathbf{u} = 0, \tag{2}$$

$$m\left(\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}.\nabla)\,\mathbf{v}\right) = K(\mathbf{u}-\mathbf{v}), \tag{3}$$

$$\frac{\partial N}{\partial t}+\operatorname{div} N\mathbf{v} = 0. \tag{4}$$

The gas and dust velocities are $\mathbf{u}$ and $\mathbf{v}$ respectively. N is the number density of dust particles, each of mass m. K is the Stokes coefficient of resistance and p, ρ, μ, the pressure, density and viscosity of the gas. The time relaxation parameter τ is given from (3) by $\tau = m/K$. When $\tau \to 0$, (3) shows that $\mathbf{u} \to \mathbf{v}$. Substituting for $(\mathbf{u}-\mathbf{v})$ in (1) from (3) we have

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u}.\nabla)\,\mathbf{u}\right) = -\operatorname{grad} p+\mu\nabla^2\mathbf{u}-Nm\left(\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}.\nabla)\,\mathbf{v}\right). \tag{5}$$

When $\tau \to 0$ (5) becomes

$$(1+f)\left(\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u}.\nabla)\,\mathbf{u}\right) = -\frac{1}{\rho}\operatorname{grad} p+\nu\nabla^2\mathbf{u}, \tag{6}$$

where we have now introduced the mass concentration of dust $f = mN/\rho$, and $\nu = \mu/\rho$. In this limiting case when we put $\mathbf{u} = \mathbf{v}$ in (4) and use (2) we find that

$$\frac{\partial N}{\partial t}+(\mathbf{u}.\nabla)\,N = 0,$$

which means that N remains constant in the neighbourhood of any given dust or gas particles. This result of course depends on the gas being assumed to behave incompressibly from (2). The simplest case to take is one in which N is uniform and equal to N_0 in the incident flow, in which case $N = N_0$ everywhere. It then follows that $f = f_0$, a constant, in this limit. Equation (6) then represents the flow of a clean gas with uniform density $\rho(1+f_0)$ and viscosity μ. The solution for the dusty gas flow at the Reynolds number R is then equivalent to the solution for a clean gas at the increased Reynolds number $R(1+f_0)$.

For flow past a sphere in which the gas velocity U changes on the length scale of the radius a of the sphere, a perturbation on the solution for $\tau = 0$ can be obtained in terms of the small dimensionless parameter $\sigma = \tau U/a$. For spherical dust particles of radius d and density ρ_d the condition $\sigma \ll 1$ becomes

$$\tfrac{2}{9}R(\rho_d/\rho)\,(d/a)^2 \ll 1.$$

This condition does not put any great restriction on R in general. For example, with $\rho_d \approx 1$, in air, $a = 10\,\text{cm}$, $d \approx 10^{-4}\,\text{cm}$ we require $R \ll 6\times 10^7$ approximately. This condition is satisfied with $U = 10^3\,\text{cm/sec}$ say in which case $R \approx 10^5$. It is thus appropriate to consider the perturbation of a flow past a sphere at large Reynolds numbers. Also it is easily verified that the neglect of dust sedimentation due to gravity in equation (3) is justified in these circumstances, in the sense that the terminal velocity of free fall of particles is much smaller than U.

We consider now the potential flow past a sphere as the limiting case when $\tau = 0$, neglecting for the present viscous boundary layers and separation effects. For this solution $\nabla^2\mathbf{u} = 0$ in equation (6) and the effect of the dust is simply to scale up the pressure variations over the sphere by the factor $(1+f_0)$. This of course leaves the drag on the sphere zero in this approximation. Let $\mathbf{u}_0$ represent the unperturbed velocity of the dust and gas, where

$$\mathbf{u}_0 = \operatorname{grad}\phi$$

and

$$\phi = U\left(r+\frac{a^3}{2r^2}\right)\cos\theta,$$

r, θ being spherical polar co-ordinates from the centre of the sphere, with $\theta = 0$ as the downstream direction, and U the mainstream velocity.

In the perturbation let $\mathbf{u} = \mathbf{u}_0+\mathbf{u}'$, $\mathbf{v} = \mathbf{u}_0+\mathbf{v}'$ represent the gas and dust velocities for a small non-zero value of τ, where $\mathbf{u}'$, $\mathbf{v}'$ represent small perturbation velocities of order τ. Also we suppose $N = N_0+N'$, $f = f_0+f'$ and $p = p_0+p'$. Neglecting the internal effect of viscosity in the gas we have from (5)

$$(\mathbf{u}_0+\mathbf{u}'.\nabla)\,\mathbf{u}_0+\mathbf{u}'+(f_0+f')\,(\mathbf{u}_0+\mathbf{v}'.\nabla)\,\mathbf{u}_0+\mathbf{v}' = -(1/\rho)\operatorname{grad}(p_0+p').$$

First-order terms alone then give

$$(\mathbf{u}_0.\nabla)\,\mathbf{u}'+(\mathbf{u}'.\nabla)\,\mathbf{u}_0+f_0\{(\mathbf{v}'.\nabla)\,\mathbf{u}_0+(\mathbf{u}_0.\nabla)\,\mathbf{v}'\}+f'(\mathbf{u}_0.\nabla)\,\mathbf{u}_0 = -(1/\rho)\operatorname{grad}p',$$

or if we write $\mathbf{w}' = \mathbf{u}'+f_0\mathbf{v}'$,

$$(\mathbf{u}_0.\nabla)\,\mathbf{w}'+(\mathbf{w}'.\nabla)\,\mathbf{u}_0+f'(\mathbf{u}_0.\nabla)\,\mathbf{u}_0 = -(1/\rho)\operatorname{grad}p'. \tag{7}$$

Similarly the linearized form of equation (3) for the dust flow is

$$\tau(\mathbf{u}_0.\nabla)\,\mathbf{u}_0 = \mathbf{u}'-\mathbf{v}'. \tag{8}$$

Further, from equation (4) we have

$$\operatorname{div}(f_0+f'.\mathbf{u}_0+\mathbf{v}') = 0,$$

which becomes in the first order

$$f_0\operatorname{div}\mathbf{v}'+(\mathbf{u}_0.\nabla)f' = 0, \tag{9}$$

using (2).

We can deduce an equation for f' from (8) and (9) by eliminating $\mathbf{v}'$ and we find

$$(\mathbf{u}_0.\nabla)f' = f_0\tau\operatorname{div}(\mathbf{u}_0.\nabla)\,\mathbf{u}_0,$$

and since $(\mathbf{u}_0.\nabla)\,\mathbf{u}_0 = \operatorname{grad}\tfrac{1}{2}\mathbf{u}_0^2$ we have

$$(\mathbf{u}_0.\nabla)f' = f_0\tau\nabla^2\tfrac{1}{2}\mathbf{u}_0^2. \tag{10}$$

Assuming that we have found f' from (10) we may obtain $\mathbf{u}'$ and $\mathbf{v}'$ as follows. Since $\operatorname{div}\mathbf{u}' = 0$ write $\mathbf{u}' = \operatorname{curl}\mathbf{A}$, where $\mathbf{A}$ has one component $A(r,\theta)$ perpendicular to the (r,θ)-planes. Then

$$\mathbf{v}' = \operatorname{curl}\mathbf{A} - \tau(\mathbf{u}_0 . \nabla)\,\mathbf{u}_0,$$

and

$$\mathbf{w}' = (1+f_0)\operatorname{curl}\mathbf{A} - f_0\tau\operatorname{grad}\tfrac{1}{2}\mathbf{u}_0^2.$$

Equation (7) then becomes

$$(\mathbf{u}_0 . \nabla)\{(1+f_0)\operatorname{curl}\mathbf{A} - f_0\tau\operatorname{grad}\tfrac{1}{2}\mathbf{u}_0^2\} + \{(1+f_0)\operatorname{curl}\mathbf{A} - f_0\tau\operatorname{grad}\tfrac{1}{2}\mathbf{u}_0^2 . \nabla\}\,\mathbf{u}_0 + f'\operatorname{grad}\tfrac{1}{2}\mathbf{u}_0^2 = -(1/\rho)\operatorname{grad}p'. \quad (11)$$

This equation has two component equations to be solved for $A(r,\theta)$ and p', when f' is known. It may also be written in the form

$$(1+f_0)\{(\mathbf{u}_0 . \nabla)\operatorname{curl}\mathbf{A} + (\operatorname{curl}\mathbf{A} . \nabla)\,\mathbf{u}_0\} + f'\operatorname{grad}\tfrac{1}{2}\mathbf{u}_0^2 = -(1/\rho)\operatorname{grad}p'', \quad (12)$$

where $p'' = p' - f_0\tau(\mathbf{u}_0 . \operatorname{grad}\tfrac{1}{2}\mathbf{u}_0^2)$.

Returning now to equation (10) for f' we find that

$$\nabla^2\tfrac{1}{2}\mathbf{u}_0^2 = \frac{9U^2a^6}{2r^8}(\sin^2\theta + 3\cos^2\theta),$$

which is always positive, and an even function about the plane $\theta = \frac{1}{2}\pi$. Thus f' increases monotonically along a streamline and the rate of increase is symmetric about $\theta = \frac{1}{2}\pi$. Written in terms of r and θ, (10) becomes

$$\left(1-\frac{a^3}{r^3}\right)\cos\theta\frac{\partial f'}{\partial r} - \frac{1}{r}\left(1+\frac{a^3}{2r^3}\right)\sin\theta\frac{\partial f'}{\partial\theta} = \frac{9\tau f_0 Ua^6}{2r^8}(1+2\cos^2\theta). \quad (13)$$

This equation cannot be integrated in a simple form in general, but some features of the solution may easily be deduced. Along the axis of symmetry upstream, where $\theta = \pi$ the equation can be integrated to give

$$f' = \frac{27\tau f_0 U}{2a}\left\{\frac{1}{\sqrt{3}}\left[\frac{\pi}{2} - \tan^{-1}\frac{2r+1}{\sqrt{3}}\right] - \tfrac{1}{6}\log\frac{(r-1)^2}{r^2+r+1} - \frac{1}{r} - \frac{1}{4r^4}\right\}, \quad (14)$$

where we have now written r for r/a. This shows that as $r \to 1$, f' increases to infinity logarithmically, i.e. as the front stagnation point is approached. Also since from equation (10) f' increases in the direction of $\mathbf{u}_0$, this singularity is continued around the surface of the sphere for all values of θ. This can be seen in mathematical terms by writing

$$f' = -\left(\frac{9\tau f_0 U}{2a}\right)\log(r-1) + A(\theta) + O\left(1-\frac{1}{r}\right),$$

in which the first term represents the logarithmic singularity at $\theta = \pi$. Substitution into equation (13) gives

$$\left(1-\frac{1}{r^3}\right)\cos\theta\left\{-\frac{9\tau f_0 U}{2a(r-1)} + \ldots\right\} - \frac{1}{r}\left(1+\frac{1}{2r^3}\right)\sin\theta\frac{dA}{d\theta} = \frac{9\tau f_0 U}{2ar^8}(1+2\cos^2\theta).$$

After cancelling $(r-1)$ in the first term this equation for $A(\theta)$ becomes at $r = 1$,

$$\frac{dA}{d\theta} = -\frac{3\tau f_0 U\{1+3\cos\theta+2\cos^2\theta\}}{a\sin\theta}.$$

Thus

$$f' = -\left(\frac{9\tau f_0 U}{2a}\right)\log(r-1) - \frac{3\tau f_0 U}{a}\{2\cos\theta + 6\log\sin\tfrac{1}{2}\theta\} + O\left(1-\frac{1}{r}\right) + \text{constant},$$

and the constant of integration may be used to match the value of f' at $\theta = \pi$. Comparing with (14) we find

$$f' = -\frac{9\tau f_0 U}{2a}\left[\log(r-1) + \tfrac{8}{3}\cos^2\tfrac{1}{2}\theta + 4\log\sin\tfrac{1}{2}\theta - \frac{\pi}{2\sqrt{3}} - \tfrac{1}{2}\log 3 + \tfrac{15}{4}\right].$$

To obtain a numerical solution for f' the author is indebted to Miss S. M. Burrough who has performed a numerical integration of equation (10) along some specific streamlines.

In dimensionless form the unperturbed streamlines are given by

$$\left(r^2 - \frac{1}{r}\right)\sin^2\theta = k, \tag{15}$$

and (10) may be written

$$u_0\frac{\partial f'}{\partial s} = f_0\tau\nabla^2\tfrac{1}{2}\mathbf{u}_0^2,$$

where $\partial f'/\partial s$ represents the rate of change of f' with length along a streamline. Using (15) to eliminate θ we may deduce the following expressions for $\partial\bar{f}/\partial r$ and $\partial\bar{f}/\partial\theta$ on the streamline k, where $\bar{f} = 2af'/9f_0\tau U$,

$$\frac{\partial\bar{f}}{\partial r} = \pm\frac{3 - \dfrac{2kr}{r^3-1}}{r^8\left(1-\dfrac{1}{r^3}\right)\left(1-\dfrac{kr}{r^3-1}\right)^{\frac{1}{2}}},$$

$$\frac{\partial\bar{f}}{\partial\theta} = -\frac{\left(3-\dfrac{2kr}{r^3-1}\right)\left(1-\dfrac{1}{r^3}\right)^{\frac{1}{2}}}{k^{\frac{1}{2}}r^6(1+\frac{1}{2}r^3)}.$$

In the expression for $\partial\bar{f}/\partial r$ the negative sign is taken from $\theta = \pi$ to $\theta = \frac{1}{2}\pi$, and the positive sign from $\theta = \frac{1}{2}\pi$ to $\theta = 0$. By numerical integration using Simpson's Rule, values of $\bar{f}$ were obtained at points along the streamlines given by $k = 0{\cdot}111$, $0{\cdot}16$, $0{\cdot}25$, $0{\cdot}36$, $0{\cdot}4489$, $0{\cdot}4624$, $0{\cdot}5$, $0{\cdot}6084$, $0{\cdot}64$, $0{\cdot}9025$, $0{\cdot}9409$, $1{\cdot}0$. The integration procedure is complicated by the fact that both the above formulae need to be used on each streamline. In the neighbourhood of $\theta = \frac{1}{2}\pi$, $\partial\bar{f}/\partial\theta$ must be used since $\partial\bar{f}/\partial r$ becomes infinite there. Near $\theta = 0$ and $\theta = \pi$, at large distances from the sphere, the $\partial\bar{f}/\partial\theta$ formula becomes unsuitable because equal intervals in θ represent increasingly large distances along the streamlines. The numerical integration was carried out in the range $0 < \frac{1}{2}\pi$, the two separate integrations being joined in the neighbourhood of $\theta = \frac{1}{4}\pi$. The integration from $\theta = \frac{1}{2}\pi$ to $\theta = \pi$ follows easily from the results for $0 < \theta < \frac{1}{2}\pi$. The author has available tables of values of $\bar{f}$ along each of the selected streamlines, but here the results are only summarized in figure 1, in which contour lines of constant $\bar{f}$ are shown. Figure 2 gives the ultimate value of $\bar{f}$ downstream as a function of k.

We notice that the dust concentration is increased on all streamlines, and this appears to conflict with the conservation of dust particles. If u' is the perturbed gas velocity downstream parallel to the axis of symmetry we have

$$2\pi \int_{y=0}^{\infty} y(U+u')(f_0+f')\,dy = 2\pi \int_0^{\infty} yUf_0\,dy,$$

where y is the distance from the axis. Since also $\int_0^{\infty} yu'\,dy = 0$ from the equation of continuity of the gas we find to the first order in the perturbation quantities

$$\int_{y=0}^{\infty} yf'\,dy = 0,$$

which is in contradiction with the result that $f' > 0$ downstream on each streamline. This points to an interesting feature of this solution, namely that dust

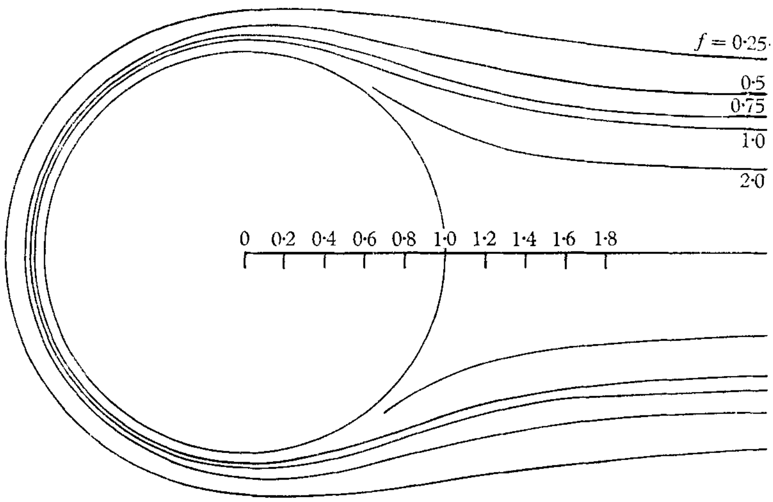

FIGURE 1. Contour lines of constant $\bar{f}$.

streamlines separate from the sphere at the front stagnation point and leave a thin dust-free boundary layer adjacent to the sphere whose thickness is of order τ. This layer approaches an asymptotic radius from the axis downstream represented by $k = k_0$. The value of k_0 can be found by equating the excess dust in the mainstream, represented in figure 2, to the loss of the mean dust concentration $\pi k_0 f_0 a^2$ in the wake $0 < k < k_0$. The equation for k_0 becomes

$$k_0 = \frac{9\sigma}{2} \int_{k_0}^{\infty} \bar{f}\,dk. \tag{16}$$

We may obtain further confirmation of the dust separation when we examine the boundary conditions at the sphere. Clearly we must have $u'_r = 0$, at $r = a$; it follows from (8) that

$$v'_r = -\tau \frac{\partial}{\partial r}(\tfrac{1}{2}\mathbf{u}_0^2) \quad \text{when} \quad r = a.$$

Thus at $r = a$, $v'_r = \frac{9}{4}\sigma U \sin^2\theta$, which is $\geqslant 0$. Further, since $(\mathbf{u}_0.\hat{\mathbf{r}}) = 0$ at $r = a$ we find $v_r > 0$ at $r = a$, except when $\theta = 0$, and π.

Ruling out the case in which the sphere acts as a steady source of dust, we must conclude that there is a separating streamline for the dust, which starts at the front stagnation point. In the first approximation the position of this separation line will be given by the equation $(\mathbf{v}.\hat{\mathbf{n}}) = 0$, or

$$(\mathbf{u}_0 + \mathbf{u}' - \tau(\mathbf{u}_0.\nabla)\,\mathbf{u}_0.\hat{\mathbf{n}}) = 0, \tag{17}$$

where $\hat{\mathbf{n}}$ is the normal to the separation line.

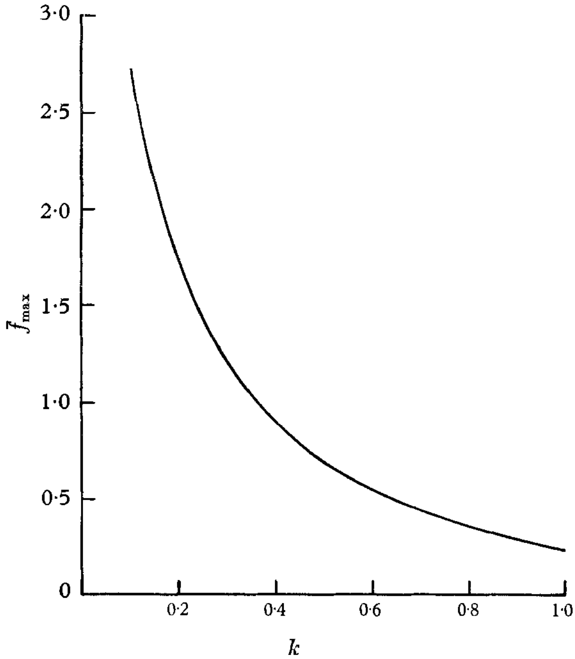

FIGURE 2. Dust concentration downstream.

In order to find this line it would be necessary to obtain $\mathbf{u}'$ from equation (11). It requires a substantial numerical integration to find $A(r, \theta)$, since f' is only known in numerical form, and the author has not attempted this. However, it is of interest to note a few more points on the structure of the solution.

It is clear that a solution of this form cannot be strictly valid as a linear perturbation in τ. The existence of a separation line avoids the difficulty of having a logarithmically infinite value of f' at $r = a$, other than at the stagnation point. Nevertheless, if a linearized solution is to be valid in which the thickness of the layer is of order τ equation (14) suggests that the value of f' on the separation line will be small only to the order $\tau \log \tau$. In fact the appearance of the logarithmic singularity in f' at the sphere stems from the process of linearization in which equation (10) for f' has become inhomogeneous. Later in the discussion, in considering the more restrictive case when f_0 is small, we are able to solve the equation $\operatorname{div} f\mathbf{v} = 0$, without having to linearize in f, and it is then seen that f/f_0 is $O(1/\tau)^{\tau}$ in the neighbourhood of the separation line.

The equation of momentum for a dust particle traversing this line is

$$\tau(v^2/R') = (\mathbf{u}.\hat{\mathbf{n}}),$$

where R' is the radius of curvature. It is then clear that $(\mathbf{u}.\hat{\mathbf{n}})$ cannot be zero on such a line, so that the gas particles must flow through the separation line. Since there is no impulsive mechanism which could make the gas particles change their velocity suddenly it is necessary to make both tangential and normal components of $\mathbf{u}$ continuous across the line. Furthermore, the gas pressure must also be continuous across this line. If we regard the position of the separation line, $\mathbf{u}$, and p as prescribed, the problem of solving the inviscid equations of motion for the gas adjacent to the sphere appears to be overdetermined. This apparent difficulty arises because this formulation of the boundary-layer problem presupposes that the outer solutions can be found independently of the boundary-layer solution, and these solutions used to impose boundary values of $\mathbf{u}$, p and the position of the separation line on the inner solution. When we examine the problem further it is clear that this is not so. One might expect the action of the gas pressure in the outer solution to make for a solution of elliptic type. This is confirmed by an examination of equation (12). If p'' is eliminated by taking the curl, the highest-order derivatives in the equation for A are of the form $(1+f_0)(u_0.\nabla)\nabla^2 A$, showing that the equation has real and imaginary characteristics. This tells us that the solution $\mathbf{u}'$ cannot be obtained simply by forward integration from upstream, and that it will depend on boundary conditions at the sphere. Thus it appears that the inner and outer solutions are coupled to each other, the solution in each region being dependent on that in the other.

To conclude this section we note that we can now establish a condition, *a posteriori*, for the validity of the Stokes flow assumption for the motion of dust particles relative to the gas. Equation (8) shows that the relative velocity is of order σU. It is therefore necessary that $\sigma U d/\nu \ll 1$, which gives $R^2 \ll \frac{9}{2}(\rho/\rho_d)(a/d)^3$. For the numerical values given previously this requires $R^2 \ll 6\times 10^{12}$ approximately.

3. Small values of f_0

The difficulties outlined above do not appear if we make the further restriction that f_0 is small. Equation (9) then shows that f' is small of the second order, and to the first-order equation (7) tells us that $\mathbf{u}' = p' = 0$. Equation (8) gives

$$\mathbf{v}' = -\tau(\mathbf{u}_0.\nabla)\,\mathbf{u}_0,$$

so that

$$\mathbf{v} = \mathbf{u}_0+\mathbf{v}' = \operatorname{grad}(\phi-\tfrac{1}{2}\tau(\operatorname{grad}\phi)^2).$$

Thus $\mathbf{v}$ remains a potential field in this case with potential

$$\Phi = Ua\left[\left(1+\frac{1}{2r^2}\right)\cos\theta-\tfrac{1}{2}\sigma\left\{\left(1-\frac{1}{r^3}\right)^2\cos^2\theta+\left(1+\frac{1}{2r^3}\right)^2\sin^2\theta\right\}\right].$$

The equation for dust streamlines is now

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{\left(1-\dfrac{1}{r^3}\right)\cos\theta-\sigma\left\{\left(1-\dfrac{1}{r^3}\right)\dfrac{3\cos^2\theta}{r^4}-\left(1+\dfrac{1}{2r^3}\right)\dfrac{3\sin^2\theta}{2r^4}\right\}}{-\left(1+\dfrac{1}{2r^3}\right)\sin\theta+\dfrac{\sigma}{r}\left\{\left(1-\dfrac{1}{r^3}\right)^2-\left(1+\dfrac{1}{2r^3}\right)^2\right\}\sin\theta\cos\theta}. \tag{18}$$

When $\sigma = 0$ this equation integrates to equation (15) and it is interesting to trace the divergence of the gas particles from the paths given by (15). In order to do so we write the equation of the streamline in the form

$$(r^2 - 1/r)\sin^2\theta = k + k'(\theta),$$

where $k'(\theta)$ is a small change in k, of order σ, representing the displacement of the particles at the angle θ. We then have

$$2\left(r^2 - \frac{1}{r}\right)\sin\theta\cos\theta + \left(2r + \frac{1}{r^2}\right)\sin^2\theta\frac{dr}{d\theta} = \frac{dk'}{d\theta}, \tag{19}$$

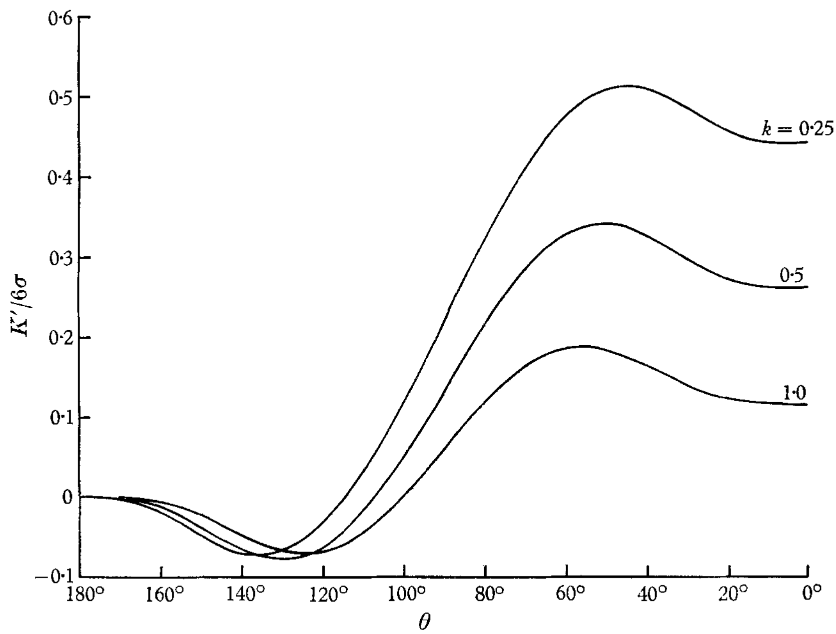

FIGURE 3. Displacement of dust particles from initial streamlines.

and by eliminating $(dr/d\theta)$ between (18) and (19) we find the following equation for $k'(\theta)$ to the first power in σ

$$\frac{dk'}{d\theta} = 2\sigma\sin\theta\left\{\frac{3k\cot^2\theta}{r^4} - \left(1+\frac{1}{2r^3}\right)\left(\frac{3}{2r^2}\right)\sin^2\theta\right\} + \frac{2\sigma k\cos^2\theta}{\sin\theta(1+\frac{1}{2}r^3)}\left\{\frac{3}{r^4} - \frac{3}{4r^7}\right\}. \tag{20}$$

With the assistance of Miss S. M. Burrough the author has integrated equation (20), along the three streamlines $k = 0{\cdot}25$, $0{\cdot}5$, $1{\cdot}0$, and the values of k' are shown in figure 3. As the sphere is approached from upstream k' becomes initially negative, showing that the dust stream does not immediately respond to the curving of the gas streamlines as they divide past the sphere. When the dust flow is deflected the diagram illustrates that it moves permanently to the outside of its initial streamline, thus increasing the value of f downstream.

In this special case it is easy to solve equation (17) for the dust separation line since now $\mathbf{u}' = 0$. If we write $r = 1+\sigma\delta(\theta)$ as the equation for this line we have, to the first order,

$$(\mathbf{u}_0 . \hat{\mathbf{n}})_{r=1+\sigma\delta} - \frac{\tau}{a} u_0^2(\theta)_{r=1} = 0,$$

which gives

$$\sin\theta \frac{d\delta}{d\theta} + 2\cos\theta . \delta = -\tfrac{3}{2}\sin^2\theta.$$

The appropriate solution of the equation, which makes $\delta = 0$ at $\theta = \pi$, is

$$\delta = \frac{3}{2\sin^2\theta}\left(\cos\theta - \frac{\cos^3\theta}{3} + \frac{2}{3}\right).$$

It is easily seen that this separation line is tangential to the sphere at $\theta = \pi$, diverges monotonically from the sphere as $\theta \to 0$, and ultimately trails along the downstream axis $\theta = 0$.

It is noticeable that in this case no formal difficulty arises inside the boundary layer, since the solution $\mathbf{u} = \mathbf{u}_0$ continues up to the surface of the sphere. The equation (10) for f' remains unaltered, and makes f' logarithmically infinite in the second order at $r = a$. But in this case an improved discussion of the distribution of f is possible. Since $\mathbf{v}$ is now explicitly known to the first order in τ the equation for f can be solved in the form

$$\operatorname{div}(f_0\{\mathbf{u}_0 - \tau(\mathbf{u}_0 . \nabla)\mathbf{u}_0\}) = 0,$$

or

$$\left[\left(1-\frac{1}{r^3}\right)\cos\theta + O(\tau)\right]\frac{\partial f}{\partial r} - \left[\frac{1}{r}\left(1+\frac{1}{2r^3}\right)\sin\theta + O(\tau)\right]\frac{\partial f}{\partial \theta} = \frac{9\sigma}{2r^8}(1+2\cos^2\theta)f.$$

We examine the solution of this equation in the neighbourhood of the separation line where $r-1$ is $O(\sigma)$. Since $1-r^{-3}$ is $O(\sigma)$ in this neighbourhood we need to retain terms of order σ arising from the term $-\tau(\mathbf{u}_0 . \nabla)\mathbf{u}_0$ of $\mathbf{v}$ in the coefficient of $\partial f/\partial r$. The equation is then

$$(x+\tfrac{3}{4}\sin^2\theta + O(\sigma))\cos\theta\frac{\partial f}{\partial x} - \tfrac{1}{2}(1+O(\sigma))\sin\theta\frac{\partial f}{\partial \theta} = \frac{3\sigma}{2}(1+2\cos^2\theta)(1+O(\sigma))f,$$

where $r = 1+\sigma x$.

We consider the equation

$$(x+\tfrac{3}{4}\sin^2\theta)\cos\theta\frac{\partial f}{\partial x} - \tfrac{1}{2}\sin\theta\frac{\partial f}{\partial \theta} = \frac{3\sigma}{2}(1+2\cos^2\theta)f.$$

With $h = \log f$ and $y = x+g(\theta)$ we find

$$\frac{\partial h}{\partial y}\left\{(y-g+\tfrac{3}{4}\sin^2\theta)\cos\theta - \tfrac{1}{2}\sin\theta\frac{dg}{d\theta}\right\} - \tfrac{1}{2}\sin\theta\frac{\partial h}{\partial \theta} = \frac{3\sigma}{2}(1+2\cos^2\theta),$$

and we choose $g(\theta)$ to make

$$(-g+\tfrac{3}{4}\sin^2\theta)\cos\theta - \tfrac{1}{2}\sin\theta(dg/d\theta) = 0,$$

namely $g = \tfrac{3}{8}\sin^2\theta$.

The equation is then

$$y\frac{\partial h}{\partial y}\cos\theta - \tfrac{1}{2}\sin\theta\frac{\partial h}{\partial \theta} = \frac{3\sigma}{2}(1+2\cos^2\theta),$$

and can be solved as previously by writing

$$h = l(\theta) + m,$$

where
$$\sin\theta \frac{dl}{d\theta} + 3\sigma(1 + 2\cos^2\theta) = 0,$$

which gives
$$l(\theta) = -6\sigma\cos\theta + \frac{9\sigma}{2}\log\left(\frac{1+\cos\theta}{1-\cos\theta}\right) + \beta.$$

The equation for m is

$$y\frac{\partial m}{\partial y}\cos\theta - \tfrac{1}{2}\sin\theta\frac{\partial m}{\partial \theta} = 0,$$

which means that m is a function of $y\sin^2\theta$.

Thus

$$f = A\left(\frac{1+\cos\theta}{1-\cos\theta}\right)^{9\sigma/2}\exp\{-6\sigma\cos\theta + F(y\sin^2\theta)\}$$

$$= A\left(\frac{1+\cos\theta}{1-\cos\theta}\right)^{9\sigma/2}\exp\{-6\sigma\cos\theta + F(x\sin^2\theta + \tfrac{3}{8}\sin^4\theta)\}. \quad (21)$$

In order to decide the scale constant A and the function F it is necessary to compare (21) with the solution integrated from infinity on the axis $\theta = \pi$. Here we find

$$\frac{1}{f}\frac{df}{dr} = \frac{-27\sigma}{2r^5(r^3-1)} + O(\sigma^2).$$

On integrating with the condition $f = f_0$ at $r = \infty$, we find

$$f = f_0\left(\frac{3}{\sigma^2 x^2}\right)^{9\sigma/4}\left\{1 - \frac{9\sigma}{2}\left(\frac{15}{4} - \frac{\sqrt{3}\pi}{6}\right) + O(\sigma^2)\right\}. \quad (22)$$

Comparison of (21) and (22) shows that to reduce (21) to a power of x we need $F(x\sin^2\theta + \frac{3}{8}\sin^4\theta)$ to be of the form $\log(x\sin^2\theta + \frac{3}{8}\sin^4\theta)$, in which case (21) becomes

$$f = A\left(\frac{1+\cos\theta}{1-\cos\theta}\right)^{9\sigma/2}(x\sin^2\theta + \tfrac{3}{8}\sin^4\theta)^{\alpha}\exp(-6\sigma\cos\theta).$$

Clearly $\alpha = -9\sigma/2$, and

$$f = A[2(1-\cos\theta)\sin^2\tfrac{1}{2}\theta]^{-9\sigma/2}(x + \tfrac{3}{8}\sin^2\theta)^{-9\sigma/2}\exp(-6\sigma\cos\theta).$$

Finally, at $\theta = \pi$ we find A given by

$$A = f_0 4^{9\sigma/2}\left(\frac{3}{\sigma^2}\right)^{9\sigma/4}\left\{1 - \frac{9\sigma}{2}\left(\frac{15}{4} - \frac{\sqrt{3}\pi}{6}\right) + O(\sigma^2)\right\}\exp(6\sigma),$$

and
$$f = f_0\left(\frac{12}{\sigma^2}\right)^{9\sigma/4}\left\{1 - \frac{9\sigma}{2}\left(\frac{15}{4} - \frac{\sqrt{3}\pi}{6}\right) + O(\sigma^2)\right\}$$
$$\times [(1-\cos\theta)\sin^2\tfrac{1}{2}\theta]^{-9\sigma/2}(x + \tfrac{3}{8}\sin^2\theta)^{-9\sigma/2}\exp\{-6\sigma(1+\cos\theta)\},$$

Thus f/f_0 is $O(\sigma^{-9\sigma/2})$ in the neighbourhood of the separation streamline.

4. The critical value of σ

Although the main discussion of this paper is based on small values of σ, it is worthwhile to digress a little in order to make note of the critical value of σ at which particles begin to collide with the sphere. This can be done on the assumptions of § 3 that the gas velocity is unchanged by the dust, and that head-on collisions with the sphere by particles on the upstream axis will be the first to occur.

The equation of motion for a particle on this axis is

$$\frac{dv}{dr} = -\frac{v + \{1 - (1/r^3)\}}{\sigma v},$$

where $v = v_r/U$. We need to solve this equation with the boundary condition $v = -1$, at $r = +\infty$.

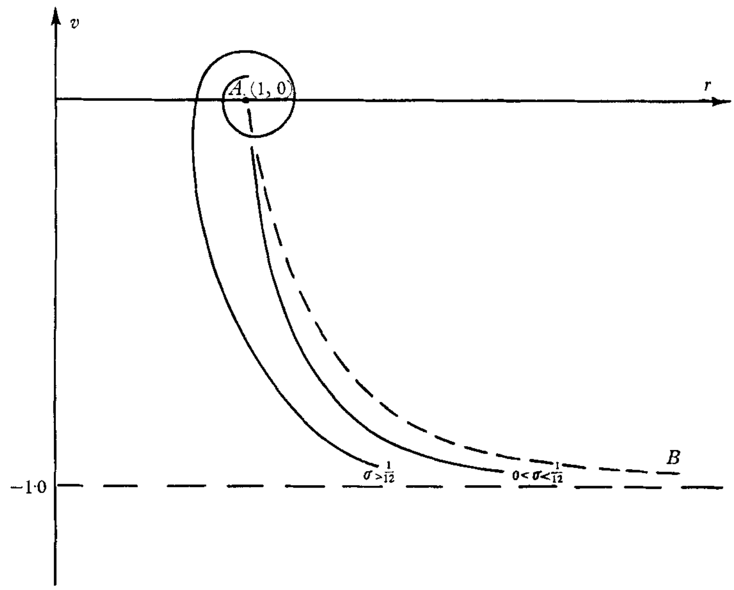

FIGURE 4. Sketch graph of particle paths on the upstream axis of symmetry, $\theta = 180°$.

In the (r, v)-plane (see figure 4) the equation has a singularity at the point $A(1, 0)$. The broken line AB represents the curve $v + (1 - 1/r^3) = 0$ which is the path of the gas particles. The behaviour of the solution near the front stagnation point is decided by the form of the singularity at A. To elucidate this write $r = 1 + h$, where h is small. The equation becomes

$$\frac{dv}{dh} = -\frac{v + 3h}{\sigma v},$$

neglecting square powers of h. This may be written in parametric form with the parameter t proportional to the time,

$$dv/dt = -(v + 3h),$$
$$dh/dt = \sigma v.$$

Thus v and h have the form $e^{\lambda t}$, where $\lambda^2 + \lambda + 3\sigma = 0$.

When $\sigma \leqslant \frac{1}{12}$ the roots λ_1 and λ_2 are real and negative. The singularity is then a node, and the time taken for particles to come to the stagnation point approaches infinity like $\log h$, as $h \to 0$. When $\sigma > \frac{1}{12}$, λ_1 and λ_2 are complex conjugate and the point A is then a spiral point. The particle paths are sketched in figure 4 in each case. When $\sigma > \frac{1}{12}$ we find v non-zero at $h = 0$ and the particles collide with the sphere in a finite time. This result is in agreement with those of Langmuir & Blodgett (1946) who have also studied numerically the trajectories of particles.

We should note here that despite the neglect of gravity in general it will influence the movement of a particle near the stagnation point in cases such as that discussed here in which the particle velocity becomes small. The general effect, in the case in which the sphere moves vertically can easily be seen. If the terminal velocity of free fall is denoted by ϵU, where ϵ is small, the equation of motion becomes

$$\sigma v \frac{dv}{dr} = -[v + (1 \pm \epsilon) - 1/r^3],$$

the negative sign being used when the sphere moves vertically downwards and the positive sign for the sphere moving upwards. In the latter case the singular point lies within the sphere and the effect of gravity will be to bring the particle on to the sphere in a finite time, irrespective of the value of σ. In the former case the singular point is outside the sphere where $r = 1 + (\epsilon/3) + O(\epsilon^2)$. When

$$\sigma < \sigma_0 = \frac{1}{12}\left(1 + \frac{4\epsilon}{3} + O(\epsilon^2)\right),$$

the particle will approach this point from upstream as $t \to \infty$. There will now be a critical value of σ, $\sigma_c > \sigma_0$. When $\sigma_0 < \sigma < \sigma_c$ the particle spirals into the critical point in the phase plane, without collision, and when $\sigma > \sigma_c$ it collides with the sphere.†

5. The drag on the the sphere

It is of mathematical interest to note that we can work out exactly the drag force on the sphere according to the linearized theory in and for the unseparated potential flow, although it will be substantially changed in practice by separation of the flow.

The rate of increase of kinetic energy is

$$\int_{y=0}^{\infty} [\tfrac{1}{2}\rho(U+u')^3 + \tfrac{1}{2}\rho(f_0+f')(U+u')^3 - \tfrac{1}{2}\rho U^3 - \tfrac{1}{2}\rho f_0 U^3]\, 2\pi y\, dy,$$

which to the first order is

$$\pi\rho U^3 \int_0^{\infty} f' y\, dy + 3\pi\rho U^2(1+f_0)\int_0^{\infty} u' y\, dy = 0.$$

Hence the drag force to the first order is accounted for solely by the rate of dissipation of energy caused by the motion of the dust relative to the gas. The resistive

† In a later paper the relation between σ_c and ϵ will be calculated.

force on a dust particle is $K(u'-v')$ and the loss of energy in time dt is

$$K(u'-v')^2\,dt = m\tau\{\text{grad}\,\tfrac{1}{2}\mathbf{u}_0^2\}^2\,dt.$$

The total loss of energy per particle is

$$m\tau\int_{-\infty}^{+\infty}\{\text{grad}\,\tfrac{1}{2}\mathbf{u}_0^2\}^2\,dt.$$

To the first order in τ we may regard the path of the particle in this integral as the unperturbed streamline. To evaluate it we write it in the form

$$m\tau\int_{t=-\infty}^{+\infty}\{\text{grad}\,\tfrac{1}{2}\mathbf{u}_0^2\}^2\frac{dr}{u_{0_r}}$$

$$= \frac{9\sigma}{2}mU^2\int_{r=r_1}^{\infty}\frac{\left\{\left[\frac{2}{r^2}\left(1-\frac{1}{r^3}\right)^2-3k\left(\frac{1}{r^4}-\frac{1}{2r^7}\right)\right]^2+\frac{k}{r^8}\left(r^2-\frac{1}{r}-k\right)\left(2-\frac{1}{2r^3}\right)^2\right\}dr}{r^4\{1-(1/r^3)\}^3\cos\theta},$$

along the streamline k, where r_1 is the dimensionless value of r at $\theta=\frac{1}{2}\pi$. The number rate at which particles pass between stream tubes k and $k+dk$ is $\pi a^2N_0U\,dk$. Hence substituting for $\cos\theta$ we find the rate of dissipation of energy

$$\frac{9\pi}{2}\sigma mN_0U^3\int_0^{\infty}dk\int_{r_1}^{\infty}\frac{\left\{\left[\frac{2}{r^2}\left(1-\frac{1}{r^3}\right)^2-3k\left(\frac{1}{r^4}-\frac{1}{2r^7}\right)\right]^2+\frac{k}{r^8}\left(r^2-\frac{1}{r}-k\right)\left(2-\frac{1}{2r^3}\right)^2\right\}dr}{r^4(1-1/r^3)^{\frac{5}{2}}(r^2-1/r-k)^{\frac{1}{2}}}.$$

This integral can be evaluated by interchanging the order of integration when we get

$$\frac{9\pi}{2}\sigma mN_0U^3\int_1^{\infty}dr\int_0^{(r^2-1/r)}\frac{\left\{\left[\frac{2}{r^2}\left(1-\frac{1}{r^3}\right)^2-3k\left(\frac{1}{r^4}-\frac{1}{2r^7}\right)\right]^2+\frac{k}{r^8}\left(r^2-\frac{1}{r}-k\right)\left(2-\frac{1}{2r^3}\right)^2\right\}dk}{r^4(1-1/r^3)^{\frac{5}{2}}(r^2-1/r-k)^{\frac{1}{2}}}$$

$$= \tfrac{1053}{440}\pi\sigma mN_0U^3.$$

Thus the drag on the sphere is $2{\cdot}393\pi\sigma mN_0U^2$.

6. Viscous effects

Two ways in which the gas viscosity may be expected to change the pattern of dust flow are by the separation of the gas flow and by changes associated with the viscous boundary layer. A first step towards making allowance for separation could be a modification of the irrotational solution used for the gas flow. This is left as a topic for further investigation.

When the viscous boundary layer is taken into account the length scale on which the gas velocity changes in the boundary layer is of order $a/R^{\frac{1}{2}} \ll a$. For the model to remain valid we need to be satisfied that this length scale remains larger than the size of the dust particles, since otherwise the law of force between the particles and the gas will need modification. The requirement is that $d/a \ll R^{\frac{1}{2}}$, and this is seen to be true under the conditions previously envisaged.

The next point of note is that the viscous boundary layer changes the way in which dust particles are concentrated near the front stagnation point. We can gain some understanding of this by an examination of the equation for f on the axis close to the stagnation point, using the dominant terms in the expansion for the viscous flow about the stagnation point. If we write $r = a+\eta$, where $\eta/a \ll 1$ we find

$$\left.\begin{aligned} u_r &= w\left\{\left(\frac{\eta}{a}\right)^2 + O\left(\frac{\eta}{a}\right)^3\right\}\cos\theta, \\ u_\theta &= -w\left\{\frac{\eta}{a} + O\left(\frac{\eta}{a}\right)^2\right\}\sin\theta, \end{aligned}\right\} \tag{23}$$

where w is a velocity of order U.

When f_0 is not assumed small we have from equations (8) and (9) the equation

$$(\mathbf{u}_0 . \nabla) f' = f_0 \tau \operatorname{div} (\mathbf{u}_0 . \nabla)\, \mathbf{u}_0 \quad \text{for} \quad f'.$$

Substituting from (23) we find on the axis $\theta = \pi$,

$$\frac{\partial f'}{\partial \eta} = -\frac{6f_0}{a}\left(\frac{w\tau}{a}\right)\left\{1 + O\left(\frac{\eta}{a}\right)\right\}.$$

Alternatively when f_0 is small we find from equation (21)

$$\frac{\partial f}{\partial \eta}\left\{1 + O\left(\frac{\eta}{a}\right)\right\} = -6\left(\frac{\tau w}{a}\right)\frac{f}{a}\left\{1 + O\left(\frac{\eta}{a}\right)\right\},$$

showing that $f = f_s e^{-6(\tau w/a)\eta/a}$ approximately, where f_s is the value of f at the stagnation point.

Both results show that the proportionate change in f across this region is small of the order $(\tau w/a)(\eta/a)$. We may thus expect that the maximum concentration of particles near the stagnation point to be given by the concentration at the edge of the viscous boundary layer. With $\eta \sim a/R^{\frac{1}{2}}$ we have from (14) that the maximum value of f' on the axis $\theta = \pi$ of the order $(9\tau f_0 U/4a)\log R$. With f_0 small we find from (22) the maximum f to be of the order $f_0(3R)^{9\sigma/4}\{1+O(\sigma)\}$.

The next point of interest is the effect of the viscous boundary layer on the dust-free layer of inviscid theory at points away from the stagnation region. In general terms we may reason that whereas the dust free layer is of order σa in its thickness, and the viscous boundary layer of order $a/R^{\frac{1}{2}}$, dust particles may not be expected to enter the viscous boundary layer when $\sigma R^{\frac{1}{2}} \ll 1$. Taken together with previous requirements of the solution we now have

$$1 \ll R \ll \frac{9}{2}\left(\frac{\rho}{\rho_d}\right)\left(\frac{a}{d}\right)^2 \ll R^{\frac{3}{2}}.$$

With $R = 10^5$, $\rho = 0{\cdot}0013$ for air and $\rho_d \sim 1$ we find

$$1{\cdot}7 \times 10^7 \ll (a/d)^2 \ll 5{\cdot}3 \times 10^9$$

approximately. With $a = 10$ cm the right-hand side of this inequality is not satisfied with $d \sim 10^{-4}$ cm as was previously postulated, and such particles might be expected to enter the viscous boundary layer. The condition under which particles will avoid the boundary layer may clearly be rather critical. Under the conditions postulated here, for example, only particles for which $(a/d)^2 \sim 10^9$ approximately are likely to do so.

When $\sigma R^{\frac{1}{2}} \gg 1$ the dust separation line lies outside the viscous boundary layer and the solution based on inviscid gas flow might be expected to be valid. However, some small modification is necessary, since the dust-free layer is tangential to the sphere at the stagnation point. This means that a small flux of particles which form the edge of the dust-free layer, having passed close to the stagnation point, will have entered the viscous boundary layer there. We can see by order of magnitude considerations that when f_0 is small and $\mathbf{u}' = 0$ in the boundary layer, particles which enter the viscous boundary layer will tend to move outwards towards the inviscid region again. A particle on the boundary layer at a distance η from the sphere, where $R^{\frac{1}{2}}(\eta/a) \sim 1$ will be carried along the boundary layer with a tangential velocity still of order U. Also the normal velocity of the gas tending to push the particle towards the sphere is of order $U\eta/a$. Since here $mU^2/a \gg KU\eta/a$ the situation remains qualitatively the same for such a particle as in the inviscid theory, and the particle will move towards the outside of the boundary layer under the dominant influence of the centrifugal force. The orders of magnitude of the velocities change when $R^{\frac{1}{2}}(\eta/a) \ll 1$, but the result remains the same provided that the tangential velocities approach zero like $UR^{\frac{1}{2}}(\eta/a)$ as $\eta \to 0$.

When $\sigma R^{\frac{1}{2}} \ll 1$ the dust separation line is within the viscous boundary layer and very near the sphere. Since now $mU^2/a \ll KU\eta/a$ when $R^{\frac{1}{2}}(\eta/a) \sim 1$ particles in the viscous boundary layer will move towards the sphere. We may represent the motion of one particle by the approximate equations

$$\tau\ddot{\xi} = (u-\dot{\xi}),$$
$$\tau\ddot{\eta} = -(\dot{\eta}+\beta\eta),$$

where ξ is the tangential co-ordinate of the particle. Here u is the tangential gas velocity in the boundary layer which is of order U, and $\beta\eta$ represents the inward normal velocity of the gas near the boundary. The coefficient β is positive, of the order U/a, and will vary with ξ on the scale a, and with η on the scale of the boundary layer width. However, we can see from this that a particle in the position (ξ_0, η_0) at time $t = 0$ has the position $\xi \sim \xi_0 + ut$, $\eta \sim \eta_0 e^{-\beta t}$ and that $\eta \sim \eta_0 e^{-(\xi-\xi_0)/a}$, in order of magnitude. When particles come so close to the sphere that $R^{\frac{1}{2}}(\eta/a) \ll 1$ the orders of magnitude of the tangential and normal gas velocities change to $UR^{\frac{1}{2}}(\eta/a)$ and $UR^{\frac{1}{2}}(\eta/a)^2$ respectively. This does not make any qualitative difference to the paths of the particle near the sphere, although it changes the time scale so that $\eta \to 0$ like $1/t$ instead of $e^{-\beta t}$. When $\sigma R^{\frac{1}{2}} \ll 1$ our general conclusion is therefore that under steady state conditions the viscous boundary layer prevents the formation of a dust-free layer since dust particles which pass close to the stagnation point are drawn closer to the sphere as they pass around it.

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